Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

GENERAL RULE FOR CONSTRUCTING ORNATE MAGIC SQUARES OF ORDERS $\equiv 0$ (MOD. 4).

Take a square lattice of order $4m$ and draw heavy lines at every fourth vertical bar and also at every fourth horizontal bar, thus dividing the lattice into m^2 subsquares of order 4. The "period" consists of the $4m$ natural numbers 1, 2, 3.... $4m$. Choose from these *any* two pairs of complementary numbers, that is, pairs whose sum is $4m+1$ and arrange these four numbers, four times repeated, as in a Jaina square (first type) in the left-hand square of the top row of subsquares in the large lattice. It is essential that the Jaina pattern shall contain only one complementary couplet in each of its four columns, i. e., if the two pairs are a_1 a_2 and b_1 b_2, every column must consist entirely of a's, or entirely of b's. The first Jaina type can be obtained by using the paths (1, 2), (2, 1) and the order a_1 b_1 a_2 b_2 four times repeated. This gives the square shown in Fig. 1, which fulfils the conditions. Proceed in the same way with each of the m subsquares in the top row, *using a different pair of complementaries in each subsquare.* Since the period 1, 2, 3.... $4m$ contains $2m$ complementary pairs and two pairs are used for each subsquare, it follows that when the top row of subsquares is filled up, all the $4m$ numbers will have been used.

a_1	b_1	a_2	b_2
a_2	b_2	a_1	b_1
a_1	b_1	a_2	b_2
a_2	b_2	a_1	b_1

Fig. 1.

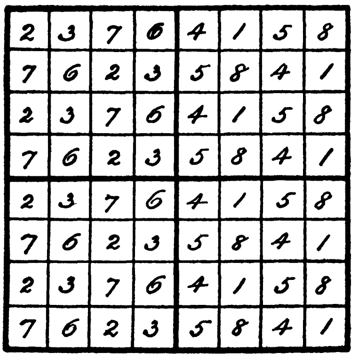
10	51	15	54	12	49	13	56
23	46	18	43	21	48	20	41
50	11	55	14	52	9	53	16
47	22	42	19	45	24	44	17
26	35	31	38	28	33	29	40
7	62	2	59	5	64	4	57
34	27	39	30	36	25	37	32
63	6	58	3	61	8	60	1

Fig. 2.

Now fill all the remaining rows of subsquares in the large lattice with replicas of the top row. The *outline* so constructed can always be turned over either of its central diagonals without repetition. The resulting square will therefore contain the first $(4m)^2$ numbers without repetition or omission, and it will always have the following magic properties.

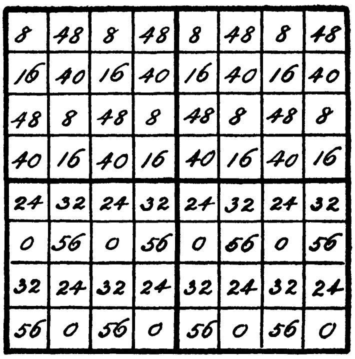
A. *The Great Square.....*

1. is magic on its $4m$ rows and $4m$ columns;
2. is pandiagonal, i. e., magic on its $8m$ diagonals;
3. has Franklin's property of *bent diagonals* in an extended sense; i. e., we can start at any cell in the top row, and proceeding downward bend the diagonal at *any* heavy horizontal bar. It matters not how many times we bend, or at which of the heavy bars, providing only that when the traverse is completed, the number of cells passed over in the one direction (downward to the right) shall be exactly equal to the number passed over in the other direction (downward to the left). Similarly we may start at any cell in the left-hand column and, proceeding diagonally to the right, bend the diagonal at *any* heavy vertical bar under the same limitations.



2	3	7	6	4	1	5	8
7	6	2	3	5	8	4	1
2	3	7	6	4	1	5	8
7	6	2	3	5	8	4	1
2	3	7	6	4	1	5	8
7	6	2	3	5	8	4	1
2	3	7	6	4	1	5	8
7	6	2	3	5	8	4	1

Fig. 3.



8	48	8	48	8	48	8	48
16	40	16	40	16	40	16	40
48	8	48	8	48	8	48	8
40	16	40	16	40	16	40	16
24	32	24	32	24	32	24	32
0	56	0	56	0	56	0	56
32	24	32	24	32	24	32	24
56	0	56	0	56	0	56	0

Fig. 4.

It will be noticed that when the order of the square is ≡ 4 (mod. 8), i. e., when m is odd, the central bars are not *heavy bars,* and also the number of rows of subsquares is odd. We cannot therefore in these cases get a magic bent diagonal traverse from top to bottom of the square, but we may stop at the last heavy bar before reaching the bottom of the square, when we shall have a sum $4(m-1)$ times the mean, or we may carry the diagonal beyond the bottom of the square and traverse the top row of subsquares a

27	46	111	106	3	58	135	94	63	22	75	130
112	105	28	45	136	93	4	57	76	129	64	21
34	39	118	99	10	51	142	87	70	15	82	123
117	100	33	40	141	88	9	52	81	124	69	16
25	48	109	108	1	60	133	96	61	24	73	132
113	104	29	44	137	92	5	56	77	128	65	20
36	37	120	97	12	49	144	85	72	13	84	121
116	101	32	41	140	89	8	53	80	125	68	17
30	43	114	103	6	55	138	91	66	19	78	127
110	107	26	47	134	95	2	59	74	131	62	23
31	42	115	102	7	54	139	90	67	18	79	126
119	98	35	38	143	86	11	50	83	122	71	14

Fig. 5. S = 870

115	110	131	158	3	78	243	190	51	94	195	174	19	46	227	222
130	159	114	111	242	191	2	79	194	175	50	95	226	223	18	47
126	99	142	147	14	67	254	179	62	83	206	163	30	35	238	211
143	146	127	98	255	178	15	66	207	162	63	82	239	210	31	34
118	107	134	155	6	75	246	187	54	91	198	171	22	43	230	219
132	157	116	109	244	189	4	77	196	173	52	93	228	221	20	45
123	102	139	150	11	70	251	182	59	86	203	166	27	38	235	214
141	148	125	100	253	180	13	68	205	164	61	84	237	212	29	36
117	108	133	156	5	76	245	188	53	92	197	172	21	44	229	220
129	160	113	112	241	192	1	80	193	176	49	96	225	224	17	48
124	101	140	149	12	69	252	181	60	85	204	165	28	37	236	213
144	145	128	97	256	177	16	65	208	161	64	81	240	209	32	33
119	106	135	154	7	74	247	186	55	90	199	170	23	42	231	218
136	153	120	105	248	185	8	73	200	169	56	89	232	217	24	41
122	103	138	151	10	71	250	183	58	87	202	167	26	39	234	215
137	152	121	104	249	184	9	72	201	168	57	88	233	216	25	40

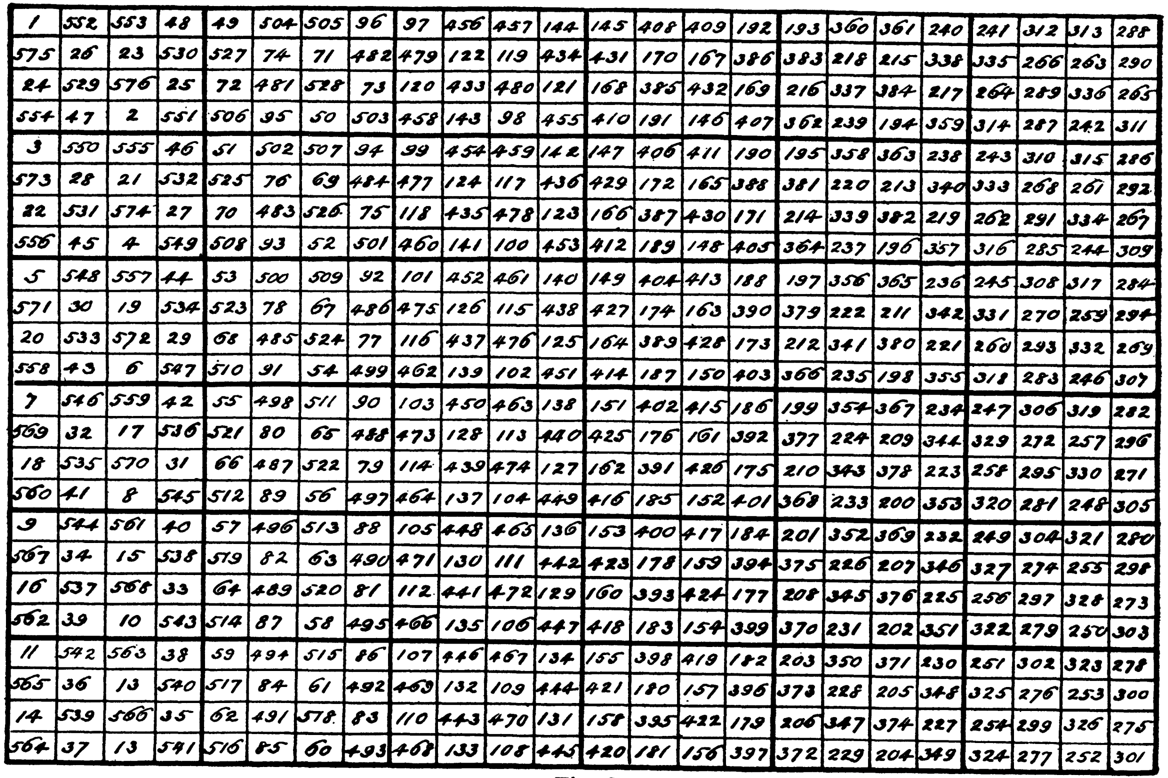
Fig. 6. S = 2056

second time, when the sum will be $4(m+1)$ times the mean. We can get in these cases a diagonal traverse $4m$ times the mean by inserting at any point one vertical series of four cells between any two heavy bars and then continuing diagonally.

4. The great square is 4-ply, and therefore 4-symmetrical, i. e., we may choose any vertical and any horizontal bar (not necessarily heavy bars) and we shall find that any four cells, symmetrically situated with regard to these two bars as axes, will contain numbers whose sum is four times the mean. It follows that any $4m$ cells which form a symmetrical figure with regard to any such axes will contain numbers whose sum is the magic sum of the great square.

1	382	20	399	3	384	18	397	5	386	16	395	7	388	14	393	9	390	12	391
40	379	21	362	38	377	23	364	36	375	25	366	34	373	27	368	32	371	29	370
381	2	400	19	383	4	398	17	385	6	396	15	387	8	394	13	389	10	392	11
380	39	361	22	378	37	363	24	376	35	365	26	374	33	367	28	372	31	369	30
41	342	60	359	43	344	58	357	45	346	56	355	47	318	54	353	49	350	52	351
80	339	61	322	78	337	63	324	76	335	65	326	74	333	67	328	72	331	69	330
341	42	360	59	343	44	358	57	345	46	356	55	347	48	354	53	340	50	352	51
340	79	321	62	338	77	323	64	336	75	325	66	334	73	327	68	332	71	329	70
81	302	100	319	83	304	98	317	85	306	96	315	87	308	94	313	89	400	92	311
120	299	101	282	118	297	103	284	116	295	105	286	114	293	107	288	112	291	109	290
301	82	320	99	303	84	318	97	305	86	316	95	307	88	314	93	309	90	312	91
300	119	281	102	298	117	283	104	296	115	285	106	294	113	287	108	292	111	289	110
121	262	140	279	123	264	138	277	125	266	136	275	127	268	134	273	129	270	132	271
160	259	141	242	158	257	143	244	156	255	145	246	154	253	147	248	152	251	149	250
261	122	280	139	263	124	278	137	265	126	276	135	267	128	274	133	269	130	272	131
260	159	241	142	258	157	243	144	256	155	245	146	254	153	247	148	252	151	249	150
161	222	180	239	163	224	178	237	165	226	176	235	167	228	174	233	169	230	172	231
200	219	181	202	198	217	183	204	196	215	185	206	194	213	187	208	192	211	189	210
221	162	240	179	228	164	238	177	225	166	236	175	227	168	234	173	229	170	232	171
220	199	201	182	218	197	203	184	216	195	205	186	214	193	207	188	212	191	209	190

Fig. 7.



1	552	553	48	49	504	505	96	97	456	457	144	145	408	409	192	193	360	361	240	241	312	313	288
575	26	23	530	527	74	71	482	479	122	119	434	431	170	167	386	383	218	215	338	335	266	263	290
24	529	576	25	72	481	528	73	120	433	480	121	168	385	432	169	216	337	384	217	264	289	336	265
554	47	2	551	506	95	50	503	458	143	98	455	410	191	146	407	362	239	194	359	314	287	242	311
3	550	555	46	51	502	507	94	99	454	459	142	147	406	411	190	195	358	363	238	243	310	315	286
573	28	21	532	525	76	69	484	477	124	117	436	429	172	165	388	381	220	213	340	333	268	261	292
22	531	574	27	70	483	526	75	118	435	478	123	166	387	430	171	214	339	382	219	262	291	334	267
556	45	4	549	508	93	52	501	460	141	100	453	412	189	148	405	364	237	196	357	316	285	244	309
5	548	557	44	53	500	509	92	101	452	461	140	149	404	413	188	197	356	365	236	245	308	317	284
571	30	19	534	523	78	67	486	475	126	115	438	427	174	163	390	379	222	211	342	331	270	259	294
20	533	572	29	68	485	524	77	116	437	476	125	164	389	428	173	212	341	380	221	260	293	332	269
558	43	6	547	510	91	54	499	462	139	102	451	414	187	150	403	366	235	198	355	318	283	246	307
7	546	559	42	55	498	511	90	103	450	463	138	151	402	415	186	199	354	367	234	247	306	319	282
569	32	17	536	521	80	65	488	473	128	113	440	425	176	161	392	377	224	209	344	329	272	257	296
18	535	570	31	66	487	522	79	114	439	474	127	162	391	426	175	210	343	378	223	258	295	330	271
560	41	8	545	512	89	56	497	464	137	104	449	416	185	152	401	368	233	200	353	320	281	248	305
9	544	561	40	57	496	513	88	105	448	465	136	153	400	417	184	201	352	369	232	249	304	321	280
567	34	15	538	519	82	63	490	471	130	111	442	423	178	159	394	375	226	207	346	327	274	255	298
16	537	568	33	64	489	520	81	112	441	472	129	160	393	424	177	208	345	376	225	256	297	328	273
562	39	10	543	514	87	58	495	466	135	106	447	418	183	154	399	370	231	202	351	322	279	250	303
11	542	563	38	59	494	515	86	107	446	467	134	155	398	419	182	203	350	371	230	251	302	323	278
565	36	13	540	517	84	61	492	469	132	109	444	421	180	157	396	373	228	205	348	325	276	253	300
14	539	566	35	62	491	578	83	110	443	470	131	158	395	422	179	206	347	374	227	254	299	326	275
564	37	13	541	516	85	60	493	468	133	108	445	420	181	156	397	372	229	204	349	324	277	252	301

Fig. 8.

B. *The Subsquares.....*

5. are balanced Jaina squares, i. e., each of them has the 36 summations of a Jaina and in each case the magic sum is four times the mean number of the great square.

6. They have the property of subsidiary minors, i. e., if we erase any *p* rows of subsquares, and any *p* columns of the same and draw the remaining rows and columns together, we have a square with *all* the properties of the original great square.

EXAMPLES

In every case the Jaina pattern quoted above is used. Fig. 2 is an example of order 8 and the complementaries have been paired thus: 2,7 with 3,6; and 4,5 with 1,8. The La Hirean primaries of Fig. 2 are shown in Figs. 3 and 4.

* * *

Fig. 5 is an example of an order 12 square in which the pairing of the complementaries is 3,10 with 4,9; 1,12 with 5,8; and 6,7 with 2,11.

1. 16 — 2. 15				13. 4 — 14. 3				12. 5 — 11. 6				8. 9 — 7. 10			
1	2	16	15	13	14	4	3	12	11	5	6	8	7	9	10
16	15	1	2	4	3	13	14	5	6	12	11	9	10	8	7
1	2	16	15	13	14	4	3	12	11	5	6	8	7	9	10
16	15	1	2	4	3	13	14	5	6	12	11	9	10	8	7

Fig. 9.

* * *

A square of order 16 is shown in Fig. 6. The couplets in this square are taken thus:

8 and 9 with 7 and 10; 1 and 16 with 5 and 12;
4 and 13 with 6 and 11; 2 and 15 with 3 and 14.

Figs. 7 and 8 show respectively squares of orders 20 and 24 in which the couplets are taken in numerical order, i. e., for order 20, 1 and 20 with 2 and 19; 3 and 18 with 4 and 17, etc.

In Fig. 8 there are 1008 magic diagonal summations. Since we can bend at any heavy bar, the number of bent diagonals from top to bottom, starting at a given cell in the top row, is the same as the number of combinations of 6 things 3 at a time, viz., 20. Therefore there are 20×24=480 bent diagonals from top to bottom and 480 more from side to side. Adding the 48 continuous diagonals we get 1008.

In the foregoing pages the question of magic knight paths has not been considered. It is, however, easy for all orders > 8 and ≡ 0 (mod. 8) to add the knight nasik property *without sacrificing any of the other features,* by a proper choice of the complementary couplets for the subsquare outlines. The example shown in Fig. 9 will explain. It shows the top row of subsquares in a scheme for order 16. The numbers above the squares indicate the couplets used, the Jaina pattern, Fig. 1, being used throughout. The rule is simple: the leading numbers, 1, 13, 12, 8 must sum four times the mean of the period, i. e., 34, while of course no one of them may be a complement of any other. Their complementaries 16,

1	32	241	240	193	224	49	48	177	176	65	96	113	112	129	160
242	239	2	31	50	47	194	223	66	95	178	175	130	159	114	111
16	17	256	225	208	209	64	33	192	161	80	81	128	97	144	145
255	226	15	18	63	34	207	210	79	82	191	162	143	146	127	98
13	20	253	228	205	212	61	36	189	164	77	84	125	100	141	148
254	227	14	19	62	35	206	211	78	83	190	163	142	147	126	99
4	29	244	237	196	221	52	45	180	173	68	93	116	109	132	157
243	238	3	30	51	46	195	222	67	94	179	174	131	158	115	110
12	21	252	229	204	213	60	37	188	165	76	85	124	101	140	149
251	230	11	22	59	38	203	214	75	86	187	166	139	150	123	102
5	28	245	236	197	220	53	44	181	172	69	92	117	108	133	156
246	235	6	27	54	43	198	219	70	91	182	171	134	155	118	107
8	25	248	233	200	217	56	41	184	169	72	89	120	105	136	153
247	234	7	26	55	42	199	218	71	90	183	170	135	154	119	106
9	24	249	232	201	216	57	40	185	168	73	88	121	104	137	152
250	231	10	23	58	39	202	215	74	87	186	167	138	151	122	103

Fig. 10.

4, 5, 9, will then have the same sum, and the second members in each square will be similarly related. The square is completed by filling the remaining rows with replicas and turning over a central diagonal. Fig. 10 is a square of order 16 constructed from the outline Fig. 9. It has all the properties of the 16^2 shown in Fig. 6, and is also magic on its 64 knight paths.

The following is an arrangement of the couplets for a square of order 24:

1.24–4.21	8.17–5.20	10.15–13.12	11.14–16.9	22.3–18.7	23.2–19.6

C. Planck.

Hayward's Heath, England.

ORNATE MAGIC SQUARES OF COMPOSITE ODD ORDERS.

When we consider these orders in the light of the general rule used for orders ≡ 0 (mod. 4) it appears at first sight that they cannot be made to fulfil all the conditions; but it is not essential to the *ply* property, nor to the balanced magic subsquares that the numbers be taken in complementary pairs for the subsquares of the outline. All that is necessary is that the groups of numbers chosen shall all have the same sum.

Suppose, as an illustration, we are dealing with order 15. If we can arrange the first 15 natural numbers in five balanced columns, three in a column, and form five magic outlines of order 3, using a different column thrice repeated for each outline, we shall have five balanced magic outlines like Fig. 1. These can be arranged in the first row of subsquares with replicas in the following rows. If we can turn this outline upon itself in some way to avoid repetitions, we shall have a magic square which will be 9-ply and with magic subsquares. But will it be pandiagonal?

2	7	15
7	15	2
15	2	7

Fig. 1.

2	6	12	11	9
15	13	8	3	1
7	5	4	10	14

Fig. 2.

In the small outlines of 9 cells made from Fig. 1 as a pattern, it